

AP Calculus AB – Unit 4 Outline – More Differentiation

Thursday 10/3	Today's Topic: Higher Order Derivatives
In-class examples:	Ex. 1 Find $f''(x)$ for $f(x) = x^{\frac{2}{3}}$ Ex. 2 Find $\frac{d^2y}{dx^2}$ for $y = x^2 + \frac{1}{x} - \sqrt{x}$ Ex. 3 For $f(x) = 3x^3 - 4x^2 + 2x - 7$, find $f^{(IV)}(x)$. Ex. 4 For $f(x) = (x^2 - 5x)^3$, find $f''(x)$.
AP Multiple Choice If $y = \left(\frac{x}{x+1}\right)^5$, then $\frac{dy}{dx} =$ (A) $5(1+x)^4$ (B) $\frac{x^4}{(x+1)^4}$ (C) $\frac{5x^4}{(x+1)^4}$ (D) $\frac{5x^4}{(x+1)^6}$ (E) $\frac{5x^4(2x+1)}{(x+1)^6}$	
Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$? (A) $y = 21x + 2$ (B) $y = 21x - 19$ (C) $y = 11x - 9$ (D) $y = 10x + 2$ (E) $y = 10x - 8$	
Homework: Worksheet 32	

Friday 10/4	Today's Topic: Linearization (Linear Approximation) and Differentials;
In-class examples:	Ex. 1 Approximate $f(3.0301)$ for $f(x) = x^2 + x$ using a tangent line approximation. Ex. 2 Use a tangent line approximation to estimate $f(9.246)$ for $f(x) = \sqrt{x}$. Ex. 3 Approximate using linearization: $\sqrt[3]{10}$
AP Multiple Choice For the function f , $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$? (A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64	
Homework: Worksheet 33	

Monday 10/7	Today's Topic: Derivatives of Sine and Cosine Functions $\frac{d}{dx}(\sin \square) = \cos \square \cdot \frac{d\square}{dx} \text{ and } \frac{d}{dx}(\cos \square) = -\sin \square \cdot \frac{d\square}{dx}$
In-Class Examples:	
Ex. 1	Find $\frac{dy}{dx}$ for: a) $y = \sin 6x$ b) $y = \cos(x^2 + 2x - 1)$
Ex. 2	Find $\frac{dy}{dx}$ for: a) $y = \frac{\cos x}{x^2}$ b) $y = x^2 \sin x$ d) $y = \sin^4 3x$
Ex. 3	Find $\frac{d^2y}{dx^2}$ for $y = \cos^2 x$
Ex. 4	Find the equation of the tangent line to $f(x) = \cos x$ when $x = \frac{\pi}{4}$.
AP Multiple Choice $\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ is (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent	
If $y = x \sin x$, then $\frac{dy}{dx} =$ (A) $\sin x + \cos x$ (B) $\sin x + x \cos x$ (C) $\sin x - x \cos x$ (D) $x(\sin x + \cos x)$ (E) $x(\sin x - \cos x)$	
Homework: Worksheet 34	

Tuesday 10/8 **Today's Topic: Derivatives of Trig Functions** – We will be finding and proving the derivatives of the remaining trigonometric functions.

1. $\frac{d(\tan \square)}{dx} = \sec^2 \square \bullet \frac{d\square}{dx}$	2. $\frac{d(\cot \square)}{dx} = -\csc^2 \square \bullet \frac{d\square}{dx}$	3. $\frac{d(\sec \square)}{dx} = \sec \square \bullet \tan \square \bullet \frac{d\square}{dx}$	4. $\frac{d(\csc \square)}{dx} = -\csc \square \bullet \cot \square \bullet \frac{d\square}{dx}$
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In-Class Examples: Find $\frac{dy}{dx}$.

Ex. 1 a) $y = \tan(5x)$ b) $y = \frac{\tan x}{x}$ c) $y = \csc(x^2)$ d) $y = \cot^3(4x)$

Ex. 2 Find the equation of the tangent line to the function $y = \sec(x)$ when $x = \frac{\pi}{4}$.

AP Multiple Choice

$$\frac{d}{dx}(x^3 \sec(2x)) =$$

- (A) $6x^2 \sec(2x) \tan(2x)$
 (B) $2x^3 \tan^2(2x) + 3x^2 \sec(2x)$
 (C) $x^3 \sec(2x) \tan(2x) + 3x^2 \sec(2x)$
 (D) $2x^3 \sec(2x) \tan(2x) + 3x^2 \sec(2x)$

$$\frac{d}{dx}(\sin^3(x^2)) =$$

- (A) $\cos^3(x^2)$ (B) $3\sin^2(x^2)$ (C) $6x\sin^2(x^2)$
 (D) $3\sin^2(x^2)\cos(x^2)$ (E) $6x\sin^2(x^2)\cos(x^2)$

If $y = \sin x \cos x$, then at $x = \frac{\pi}{3}$, $\frac{dy}{dx} =$

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

Homework: Worksheet 35

Wednesday 10/9

Today's Topic: Derivatives of logarithmic and exponential functions

$$\frac{d}{dx}(e^{\square}) = e^{\square} \cdot d\square$$

$$\frac{d}{dx}(b^{\square}) = b^{\square} \cdot d\square \cdot \ln b$$

$$\frac{d}{dx}(\ln \square) = \frac{d\square}{\square}$$

$$\frac{d}{dx}(\log_b \square) = \frac{d\square}{\square} \cdot \ln b$$

In-Class Examples: Ex. 1 Evaluate each derivative.

a. $\frac{d}{dx}[e^{2x^2-3x}]$

b. $\frac{d}{dx}[4^{\sin x}]$

c. $\frac{d}{dx}[xe^x]$

Ex. 2 Evaluate each derivative.

a. $\frac{d}{dx}[\ln(x^2+1)]$

b. $\frac{d}{dx}[\log_3(x^2-5x+1)]$

c. $\frac{d}{dx}[(\ln x)^3]$

AP Multiple Choice

$$\lim_{h \rightarrow 0} \frac{e^{(2+h)} - e^2}{h} =$$

- (A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$

What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

- (A) $-\frac{1}{e}$ (B) $-\frac{3}{4e}$ (C) $-\frac{1}{4e}$ (D) $\frac{1}{4e}$ (E) $\frac{1}{e}$

The slope of the line tangent to the graph of $y = \ln(1-x)$ at $x = -1$ is

- (A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\ln 2$ (E) 1

Homework: Worksheet 36

Thursday 10/10

Today's Topic: Quiz (Higher Order Derivatives, Linearization, Derivatives of ALL Trigs)

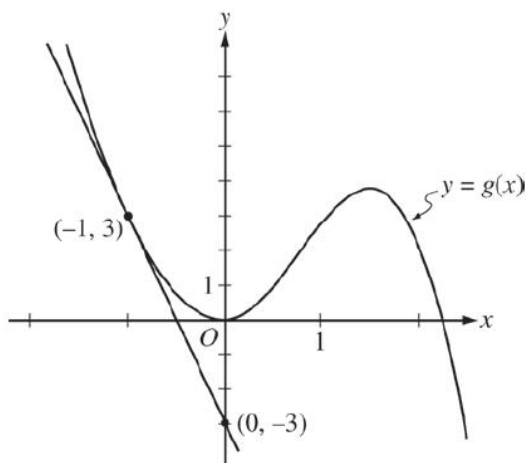
In-Class Examples: None

AP Multiple Choice



The graph of $y = e^{\tan x} - 2$ crosses the x -axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

- (A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747



The figure above shows the graph of the function g and the line tangent to the graph of g at $x = -1$. Let h be the function given by $h(x) = e^x \cdot g(x)$. What is the value of $h'(-1)$?

- (A) $\frac{9}{e}$ (B) $\frac{-3}{e}$ (C) $\frac{-6}{e}$ (D) $\frac{-6}{e} - \frac{3}{e^2}$ (E) -6

If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Homework: None

Friday 10/11	Today's Topic: L'Hopital's Rule		
In-Class Examples: Ex. 1 $\lim_{x \rightarrow -1} \frac{2x^2 - 2}{x + 1}$ Ex. 2 $\lim_{x \rightarrow \infty} \frac{3x^2 - 1}{2x^2 + 1}$ Ex. 3 $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$ Ex. 4 $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$ Ex. 5 $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 3x}$ Ex. 6 $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{-x}}$			
AP Multiple Choice $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$ is (A) $-\frac{1}{4}$ (B) 0 (C) 1 (D) $\frac{5}{4}$ (E) nonexistent			
$\lim_{x \rightarrow 0} \frac{\sin x}{e^x - 1}$ is (A) 1 (B) $\frac{1}{e}$ (C) 0 (D) nonexistent			
$\lim_{x \rightarrow 3} \frac{\tan(x - 3)}{3e^{x-3} - x}$ is (A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) nonexistent			
Homework: Worksheet 37			

Monday 10/14	Today's Topic: Implicit Differentiation	
Warm-Up: Find the second derivative of $f(x) = xe^x$		
In-Class Examples: Ex. 1 Find $\frac{dy}{dx}$ for $x^2 + y^2 = 1$. Ex. 2 Find $\frac{dy}{dx}$ for $3x^2 + 5y = 2x$ Ex. 3 Find $\frac{dy}{dx}$ for $x^2 + xy - y^3 = 7$ Ex. 4 Find $\frac{dy}{dx}$ for $\sin(xy) = 2x + 5$		
AP Multiple Choice If $y^3 + y = x^2$, then $\frac{dy}{dx} =$ (A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1 + 3y^2}$		
Homework: Worksheet 38		

Tuesday 10/15	Today's Topic: Implicit Differentiation
In-Class Examples: Ex. 1 Find the tangent and normal to the graph of $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.	
AP Multiple Choice	
If $(x + 2y) \cdot \frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?	
(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined	
If $e^{xy} - y^2 = e - 4$, then at $x = \frac{1}{2}$ and $y = 2$, $\frac{dy}{dx} =$	
(A) $\frac{e}{4}$ (B) $\frac{e}{2}$ (C) $\frac{4e}{8-e}$ (D) $\frac{4e}{4-e}$ (E) $\frac{8-4e}{e}$	
Homework: Worksheet 39	

Thursday 10/17	Today's Topic: Derivatives of Inverse Trig Functions
In-Class Examples: : Evaluate each of the following:	
1. $\frac{d}{dx}[\arcsin(2x)]$ 2. $\frac{d}{dx}[\arctan(3x)]$ 3. $\frac{d}{dx}[\arcsin \sqrt{x}]$ 4. $\frac{d}{dx}[\arctan e^{2x}]$ 5. $\frac{d}{dx}\left[\arccos\left(\frac{3}{x}\right)\right]$	
AP Multiple Choice	
$\frac{d}{dx}(\tan^{-1}x + 2\sqrt{x}) =$	
(A) $-\frac{1}{\sin^2 x} + \frac{1}{\sqrt{x}}$ (B) $\frac{1}{\sqrt{1-x^2}} - 4\sqrt[3]{x}$ (C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$ (D) $\frac{1}{1+x^2} - 4\sqrt[3]{x}$ (E) $\frac{1}{1+x^2} + \frac{1}{\sqrt{x}}$	
Homework: Worksheet 40	

Friday 10/18	Today's Topic: Review
Warm-Up: Find an equation of the tangent line to the curve of $f(x) = xe^x - e^x$ when $x = 1$.	
In-Class Examples: None	
Homework: Worksheet 41	

Monday 10/21	Today's Topic: Review
In-Class Examples: None	
Homework: Worksheet 42	

Tuesday 10/22	Today's Topic: All Derivatives Exam!
In-Class Examples: None	
Homework: None.	